HOMEWORK 5

STA5724.01, Probability
Fall Semester, 2007

Due: Friday, October 5th, 2007

1 We are interested in the probability that a patient has measles given the knowledge that they have spots:

$$Pr(\text{patient-has-measles}|\text{patient-has-spots}).$$

Sometimes we will know how likely some "evidence" is, if some hypothesis is true, but not the other way around. For example, we may know that 50% of people with measles have spots. We may also know that:

The only diseases that cause spots are measles, chickenpox and lassa fever. 60% of people with chickenpox have spots. 80% of people with lassa fever have spots. There is a 1% chance of someone in a given population having measles (given no evidence for or against). There is a 1% chance of them having chickenpox. There is a 0.05% chance of them having lassa fever. Calculate $Pr(\text{patient-has-measles}|\text{patient-has-spots})$.

2 Suppose that k evenets $B_1, B_2, \cdots, B_k$ form a partition of Ω. For $i = 1, \cdots, k$, let $Pr(B_i)$ be the prior probability of B_i. Let $A \subset \Omega$ with $Pr(A) > 0$. Let $Pr(B_i|A)$ be the posterior probability of B_i given that the event A has occured. Prove that if $Pr(B_1|A) < Pr(B_1)$, then $Pr(B_i|A) > Pr(B_i)$ for at least one value of i.

3 Suppose that a fair coin is tossed independently n times. Determine the probability of obtaining exactly $n-1$ heads given

(a) that at least $n-2$ heads are obtained and

(b) heads are obtained one the first $n-2$ tosses.

4 Show that there does not exist any number c such that the following function would be a p.f.:

$$f(x) = \begin{cases} \frac{c}{x} & \text{for } x = 1, 2, \cdots \\ 0 & \text{otherwise.} \end{cases}$$

5 Suppose that a random variable X has a uniform distribution on the interval $[-2, 8]$. Find the p.d.f. of X and the value of $Pr(0 < X < 7)$.

6 Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} c\exp(-2x) & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c and sketch the p.d.f.

(b) Find the value of $Pr(1 < X < 2)$.