HOMEWORK 6

STA5724.01, Probability
Fall Semester, 2007

Due: Friday, October 12th, 2007

1 Suppose that a point (X, Y) is chosn at random from the region S in the xy-plane containing all points (x, y) such that $x \geq 0$, $y \geq 0$ and $4y + x \leq 4$.
(a) Determine the joint p.d.f. of X and Y.
(b) Suppose that S_0 is a subset of the region S having area α and determine $Pr[(X, Y) \in S_0]$.

2 Suppose that a point (X, Y) is chosn at random from the region S defined as

$$S = \{(x, y)|0 \leq x \leq 2 \text{ and } 0 \leq y \leq 4\}.$$

(a) Determine the joint p.d.f. of X and Y, the marginal p.d.f. of X and the marginal p.d.f. of Y.
(b) Are X and Y independent?

3 Suppose that a joint p.d.f. if X and Y is

$$f(x, y) = \begin{cases} 24xy & \text{for } x \geq 0 \text{ and } x + y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent?

4 Let f_2 be the marginal p.f. of Y such that $f_2(y_0) = 0$. Let $A_0 = [y_0 - \epsilon, y_0 + \epsilon]$ and $A_1 = [y_1 - \epsilon, y_1 + \epsilon]$ with $f_2(y_1) > 0$. Assume that f_2 is continuous at y_0 and y_1. Show that

$$\lim_{\epsilon \to 0} \frac{Pr(Y \in A_0)}{Pr(Y \in A_1)} = 0.$$

5 Suppose that X and Y are independent r.v. Suppose that X has a discrete distribution concentrated on finitely many distinct values with a p.f. f_1. Suppose that Y has a continuous distribution with a p.d.f. f_2. Let $Z = X + Y$. Show that Z gas a continuous distribution and find its p.d.f.