

HOMEWORK 8
STA5724.01, Probability
Fall Semester, 2007

Due: Friday, October 26th, 2007

1 Then show that

$$F^{-1}(F(x)) \leq x, \forall x \in \mathbb{R} \text{ with } 0 < F(x) < 1,$$

and

$$F(F^{-1}(x)) \geq p, \forall p \in (0, 1).$$

2 Show that for all $x \in \mathbb{R}$ and all $p \in (0, 1)$,

$$F^{-1}(p) \leq x \text{ if and only if } F(x) \geq p.$$

3 Suppose that X has probability density function $f(x) = a/x^{a+1}$, $x \geq 1$ and $a > 0$.

- i) Find the distribution function and sketch the graph.
- ii) Sketch the graph of the probability density function.
- iii) Find the quantile function and sketch the graph.
- iiii) Find $Pr(1/2 \leq X \leq 1)$.

4 Suppose that a pair of fair dice are rolled and the sequence of scores (X_1, X_2) is recorded.

- i) Find the distribution function of $Y = X_1 + X_2$ the sum of the scores.
- ii) Find the distribution function of $V = \max\{X_1, X_2\}$, the maximum score.
- iii) Find the conditional distribution function of Y given $V = 5$.

5 Suppose (X, Y) has a density function $f(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$.

- i) Find the distribution function of (X, Y) .
- ii) Find the distribution function of X .
- iii) Find the distribution function of Y .
- iiii) Find the conditional distribution function of X given $Y = y$ for $0 < y < 1$.
- v) Find the conditional distribution function of Y given $X = x$ for $0 < x < 1$.
- iv) Are X and Y independent?