

HOMEWORK 9
STA5724.01, Probability
Fall Semester, 2007

Due: Friday, November 2nd, 2007

1 Julia collects figures from corn flakes packets. Each packet contains one figure, and n distinct figures make a complete set. Show that the expected number of packets that Julia needs to buy to collect a complete set is

$$n \sum_{i=1}^n 1/i.$$

2 Suppose that X_1, X_2, \dots, X_n are independent identically distributed positive random variables with $\mathbb{E}(X_i) = \mu < \infty$ and $\mathbb{E}(X_i^{-1}) < \infty$. Let $S_n = \sum_{i=1}^n X_i$. Show that for $m \leq n$

$$\mathbb{E}(S_m/S_n) = m/n,$$

and

$$\mathbb{E}(S_m/S_n) = 1 + (m - n)\mu\mathbb{E}(S_n^{-1}).$$

3 Suppose that the random variables X_1, X_2, \dots, X_n are independent identically distributed from a uniform distribution on the interval $[0, 1]$. Let $Y_1 = \min\{X_1, X_2, \dots, X_n\}$ and $Y_2 = \max\{X_1, X_2, \dots, X_n\}$. Find $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y_2)$. Show your work.

4 Suppose that the random variables X_1, X_2, \dots, X_n are independent identically distributed from a continuous distribution for which the d.f. is F . Let $Y_1 = \min\{X_1, X_2, \dots, X_n\}$ and $Y_2 = \max\{X_1, X_2, \dots, X_n\}$. Find $\mathbb{E}(F(Y_1))$ and $\mathbb{E}(F(Y_2))$.

5 Suppose that the random variables X_1, X_2, \dots, X_n are independent identically distributed from a continuous distribution on the real line for which the p.d.f. is f . Find the expectation of the number of observations in the sample that fall within a specific interval $a \leq x \leq b$.