

Practice 1
STA 524.01, Probability
Fall Semester, 2007

- 1 Let $A, B, C \subset \Omega$ be three events. Express in symbols the events:
- (i) only A occurs
 - (ii) all three events occur
 - (iii) at least one event occurs
 - (iv) one and only one event occurs
 - (v) no event occurs
 - (vi) not more than two events occur.

2 You should know how to count from all examples which I gave you in the class.

- 4 (i) If $A, B, C \subset \Omega$ are three events, show that

$$P(A^c \cap (B \cup C)) = P(B) + P(C) - P(B \cap C) - P(C \cap A^c) - P(A^c \cap B) + P(A^c \cap B \cap C).$$

- (ii) How many of the numbers $1, \dots, 500$ are not divisible by 7 but are divisible by 3 or 5?

6 There are 2 boxes. Each box contains equal numbers of 10 red balls and 10 blue balls. A box is chosen at random and one ball is taken out of the box. Suppose that you are allowed to rearrange blue balls between two boxes: you can put k blue balls in one box and $20 - k$ balls in the other box.

- (i) Find the probability p_k that a white ball is picked.
- (ii) What is the best strategy to guarantee that a white ball is picked?

7 A device consists of 10 components. Each of the components is working with probability p . The device is working if at least 8 of its components are working. What is the probability that the device is working?

8 Two cards are taken at random from an ordinary pack of 52 cards. Find the probabilities that:

- (i) both cards are aces (event A)
- (ii) the pair of cards includes an ace (event B)
- (iii) the pair of cards includes the ace of hearts (event C).

Show that $P(A|B) \neq P(A|C)$.

9 Examination candidates are graded into four classes known conventionally as I, II-1, II-2 and III, with probabilities $1/8, 2/8, 3/8$ and $2/8$ respectively. A candidate who misreads the rubric - a common event with probability $2/3$ - generally does worse, his probabilities being $1/10, 2/10, 4/10$ and $3/10$. What is the probability:

- (i) that a candidate who reads the rubric correctly is placed in the class II-1?
- (ii) that a candidate who is placed in the class II-1 has read the rubric correctly?

10 Parliament contains a proportion p of Conservative members, who are incapable of changing their minds about anything, and a proportion $1 - p$ of Labour members who change their minds completely at random (with probability r) between successive votes on the same issue. A randomly chosen member is noticed to have voted twice in succession in the same way. What is the probability that he will vote in the same way next time?

11 There are n people gathered in a room. What is the probability that at least one has the same birthday as you? What value of n makes this probability close to $1/2$?

- 12** Suppose that n balls are placed at random into n boxes, find the probability that there is exactly one empty box.
- 13** A fair coin is tossed until either the sequence HHH occurs, in which case A wins, or the sequence THH occurs, when B wins. Calculate the probability B wins.
- 14** An unfair coin is tossed such that the probability to see H is p and the probability to see T is $1 - p$. We toss the coin until we see H. Let p_n be the probability to see H the first time. Calculate p_n .
- 15** You should be able to prove theorems, lemmas, propositions which I showed proofs in classes. Prove all theorems, lemmas, propositions which I showed proofs in classes.
- 16** Solve the problems at http://people.hofstra.edu/Stefan_Waner/tutorialsf3/unit6_6.html
- 17** Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?
- 18** Exercises 5, 7, 9, 11, 13, 15, 17 in Section 2.2.
- 19** Exercises 1, 3, 7, 9, 11, 13, 15, 17, 19 in Section 2.3.