HOMEWORK 2

STA5724.01, Probability
Fall Semester, 2008

Due: Friday, September 19, 2008

1 For a certain style of new cars, blue, white, black and green are in equal demand. Three successive orders are placed for automobiles of this style. Find the probabilities:

a. One blue, one white and one green are ordered.

b. Two blues are ordered.

c. At least one black is ordered.

d. Exactly two of the orders are for the same color.

2 Prove that for every two events A and B, the probability that exactly one of the two events will occur is

$$Pr(A) + Pr(B) - 2Pr(AB).$$

3 For every two events A and B, show that

$$Pr(A) = Pr(AB) + Pr(AB^c).$$

4 Consider an experiment in which a fair coin is tossed once and a balanced die is rolled once.

a. Describe the sample space for this experiment.

b. What is a probability that a head will be obtained on the coin and an odd number will be obtained on the die?

c. If 12 balls are thrown at random into 20 boxes, what is the probability that no box will receive more than one ball?

5 Prove that for all positive integer n

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}.$$

6 **a.** Prove for all positive integer n

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

b. Prove for all positive integer n

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$