HOMEWORK 3

STA5724.01, Probability
Fall Semester, 2008

Due: Mon, September 29, 2009

1 For any events A, B, and C, such that $Pr(C) > 0$, prove that

$$Pr(A \cup B|C) = Pr(A|C) + Pr(B|C) - Pr(AB|C).$$

2 Assuming that A and B are independent events, prove that A^c and B^c are also independent events.

3 Suppose A, B, and C are three independent events such that $Pr(A) = 1/4$, $Pr(B) = 1/3$, $Pr(C) = 1/2$.
(a) Determine the probability that none of the three events will occur.
(b) Determine the probability that exactly one of these three events wull occur.

4 We are interested in the probability that a patient has measles given the knowledge that they have spots:

$$Pr(\text{patient-has-measles}|\text{patient-has-spots}).$$

Sometimes we will know how likely some "evidence" is, if some hypothesis is true, but not the other way around. For example, we may know that 50% of people with measles have spots. We may also know that:

The only diseases that cause spots are measles, chickenpox and lassa fever. 60% of people with chickenpox have spots. 80% of people with lassa fever have spots. There is a 1% chance of someone in a given population having measles (given no evidence for or against). There is a 1% chance of them having chickenpox. There is a 0.05% chance of them having lassa fever. Calculate $Pr(\text{patient-has-measles}|\text{patient-has-spots})$.

5 Prove the following statement:

Suppose A_1, A_2, and B are events such that $Pr(A_1B) > 0$. Then, A_1 and A_2 are conditionally independent given B if and only if $Pr(A_2|A_1B) = Pr(A_2|B)$.