

HOMEWORK 4
STA5724.01, Probability
Fall Semester, 2008

Due: Friday, October 17, 2008

1 Suppose that a random variable X has a uniform distribution on the interval $[-2, 8]$. Find the p.d.f. of X and the value of $Pr(0 < X < 7)$.

2 Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} c \exp(-2x) & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of the constant c and sketch the p.d.f.
- (b) Find the value of $Pr(1 < X < 2)$.

3 Suppose that a point (X, Y) is chosen at random from the region S in the xy -plane containing all points (x, y) such that $x \geq 0$, $y \geq 0$ and $4y + x \leq 4$.

- (a) Determine the joint p.d.f. of X and Y .
- (b) Suppose that S_0 is a subset of the region S having area α and determine $Pr[(X, Y) \in S_0]$.

4 Suppose that a point (X, Y) is chosen at random from the region S defined as

$$S = \{(x, y) | 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 4\}.$$

- (a) Determine the joint p.d.f. of X and Y , the marginal p.d.f. of X and the marginal p.d.f. of Y .
- (b) Are X and Y independent?

5 Suppose that a joint p.d.f. of X and Y is

$$f(x, y) = \begin{cases} 24xy & \text{for } x \geq 0 \text{ and } x + y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent?