HOMEWORK 5

STA5724.01, Probability
Fall Semester, 2008

Due: Friday, October 24th, 2008

1 Suppose that a point (X, Y) is chosn at random from the region S in the xy-plane containing all points (x, y) such that $x \geq 0$, $y \geq 0$ and $4y + x \leq 4$.
(a) Determine the joint p.d.f. of X and Y.
(b) Suppose that S_0 is a subset of the region S having area α and determine $P[(X, Y) \in S_0]$.

2 Suppose the joint p.d.f. of random variables X and Y is as follows:

$$f(x, y) = \begin{cases} c(x + y^2) & \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine
(a) the conditional p.d.f. of X for every given value of Y and
(b)

$$P(X < 1/2 | Y = 1/2)$$

3 Suppose that a fair coin is tossed independently n times. Determine the probability of obtaining exactly $n - 1$ heads given
(a) that at least $n - 2$ heads are obtained and
(b) heads are obtained one the first $n - 2$ tosses.

4 Suppose that a point (X, Y) is chosen at random from the circle

$$\{(x, y) | (x - 1)^2 + (y + 2)^2 \leq 9\}.$$

Determine
(a) the conditional p.d.f. of Y for every given value of X and
(b)

$$P(Y > 0 | X = 2)$$

5 Suppose n random variables $X_1, \cdots, X_n$ form a random sample from a discrete dustribution for which the p.f. is f. Determine the value of $Pr(X_1 = X_2 = \cdots = X_n)$.