HOMEWORK 7

STA5724.01, Probability
Fall Semester, 2008

Due: Friday, November 7th, 2008

1 Compute the variance of a r.v. X which is distributed uniformally on $[0, 1]$.

2 Suppose X is a r.v. with $\mathbb{E}(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$. Show that

$$\mathbb{E}[X(X-1)] = \mu(\mu-1) + \sigma^2.$$

3 Suppose random variables X and Y are independent with finite mean such that

$$\mathbb{E}(X) = \mathbb{E}(Y).$$

Show that

$$\mathbb{E}[(X-Y)^2] = Var(X) + Var(Y).$$

4 Suppose that the random variables $X_1,\ X_2, \cdots X_n$ are independent identically distributed from a uniform distribution on the interval $[0, 1]$. Let $Y_1 = \min\{X_1,\ X_2, \cdots X_n\}$ and $Y_2 = \max\{X_1,\ X_2, \cdots X_n\}$. Find $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y_2)$. Show your work.

5 Suppose that the random variables $X_1,\ X_2, \cdots X_n$ are independent identically distributed from a continuous distribution on the real line for which the p.d.f. is f. Find the expectation of the number of observations in the sample that fall within a specific interval $a \le x \le b$.