HOMEWORK 7

STA 624, 2007 Applied Stochastic Processes

Due: Friday, March 23rd, 2007.

Readings: Chapter 3.

Regular Problems

1 (Lawler, 3.2) Let X_t and Y_t be two independent Poisson processes with rate parameters λ_1 and λ_2, respectively, measuring the number of customers in store 1 and store 2, respectively.

(a) What is the probability that a customer arrives in store 1 before any customers arrive in store 2?

(b) What is the probability that in the first hour, a total of exactly four customers have arrived at the two stores?

(c) Given that exactly four customers have arrived at two stores, what is the probability that all four went to the store 1?

(d) Let T be the time of arrival of the first customer arrived at store 2. Then X_T is the number of customers in store 1 at the time of the first customer arrived at store 2. Find the probability distribution of X_T (i.e. for each k, find $P(X_T = k)$).

2 (Lawler 3.3) Suppose X_t and Y_t are two independent Poisson processes with rate parameters λ_1 and λ_2, respectively, measuring of calls arriving at two different phones. Let $Z_t = X_t + Y_t$.

(a) Show that Z_t is a poisson process. What is the rate parameter for Z?

(b) What is the probability that the first call comes on the first phone?

(c) Let T be the first time that at least one call has come from each of the two phones. Find the density and distribution function of the random variable T.

3 Arrivals of passengers at a bus stop form a Poisson process X_t with rate $\lambda = 2$ per unit time. Assume that a bus departed at time $t = 0$ leaving no customers behind. Let T denote the arrival time of the next bus. Then the number of passengers present when it arrives is X_T. Suppose that the bus arrival time T is independent of the Poisson process and that T has the uniform probability density function

$$f_T(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1, \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the conditional moments $E[X_T|T = t]$ and $E[X_T^2|T = t]$.

(b) Determine the mean $E[X_T]$ and variance $\text{Var}[X_T]$.

Computer Problem This computer problem has three parts. In the first part we will look at the Poisson process, in the second we will look at the Poisson distribution, and in the third we will examine the relationship between the Poisson distribution and the binomial distribution.

(a) The Poisson distribution is often used to model the number of events that occur in a fixed time interval or fixed spatial region when they are occurring "at random", like shooting stars.

Suppose you spend five minutes watching shooting stars, and you know that on average they fall at 3 per minute. Thus, the rate parameter for our five-minute time period is lambda=15. We expect, on average, to see 15 shooting stars per five minutes. Show what a Poisson process might look like by using a simulation by breaking that five-minute-long time interval into very small intervals of 1 second and plot them.

(b) Simulate 10,000 star-gazing five-minute periods all at once. Then estimate the count of how many shooting stars you would see in a five-minute stretch. ALso estimate its mean, standard deviation, and variance.

(c) In class we learned that Binomial(n, p) can be approximated by Poisson(λ) with $\lambda = np$, when n is very large. Let us check the approximation. Try different values of $n = 75$ and $n = 100$ with the same λ in (a). See how the Poisson approximates a binomial better when n (in the binomial) is large. Compare the Binomial(n,p) plot against to the Poisson(lambda = np) plot.