HOMEWORK 9

STA 624.01

Applied Stochastic Processes

Due: April 9th, 2007

Regular Problems

1 (Lawler 3.11) Let X_t be a countinuous-time birth-dan-death process with the birth rate $\lambda_n = 1 + (1/(n+1))$ and death rate $\nu_n = 1$. Is it process positive recurrent, null reccurent, or transient?

2 (Lawler 3.12) Consider the population model (Example 3, Section 3.3). For which value of ν and λ is extinction certain, i.e. when is the probability of reaching state 0 equal to 1?

3 (Lawler 3.14) Consider a birth-and-death process with $\lambda_n = 1/(n+1)$ and $\nu_n = 1$. Show that the process is positive recurrent and give the stationary distribution.

4 Logistic Growth Model Suppose that y_t is the population at time t. The logistic growth model assumes that the population cannot continue to grow indefinitely: there is a resource limitation that prevents the population from growing past size K. The deterministic model is:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry\left(1 - \frac{y}{K}\right).$$

In other words, the rate of births is ry and the rate of deaths is ry^2/K. Note that solutions to this deterministic model can never rise over K.

Consider the continuous time Markov chain $\{Y_t\}$ where a birth occurs at rate rY_t and a death occurs at rate rY_t^2/K, on state space $\{0, 1, \ldots, N\}$. (So the birth rate at state N is 0.)

(a) Is this chain irreducible?

(b) For $r = 0.015, K = 10, Y_0 = 10$ and $N = 20$, find the expected time until the population goes extinct.

(c) Do the same as part (a) for $N = 30$.

(d) One way to define a quasistationary distrbiution is to make the death rate 0 when $Y_t = 1$. Given this change, and $r = 0.015, K = 10, N = 20$, find the stationary distribution and plot it.

Computer Problem: M/M/k queue Suppose you spend five minutes observing the number of people in the queue and suppose $k = 4$. Let $\lambda = 15$ and $\nu = 10$ per 5 minutes. By breaking that five-minute-long time interval into very small intervals of 1 second, simulate the number of people in the queue. Also estimate the average time to stay the same number of people in the queue for each number 0 to 100.