HOMEWORK 0

STA624.01, Stochastic processes
Spring Semester, 2008

Due: Fri January 11th, 2008

1 For events A, B such that $A \subset B$, show that

$$B^c \subset A^c.$$

For every three events A, B, C, show that

$$A(B \cup C) = (AB) \cup (AC).$$

2 For every two events A and B, show that AB and AB^c are disjoint and also prove that

$$A = (AB) \cup (AB^c).$$

3 For every collection of events A_i for $i \in I$ (I is an index set), show that

$$(\cup_{i \in I} A_i)^c = \cap_{i \in I} A_i^c$$

and

$$(\cap_{i \in I} A_i)^c = \cup_{i \in I} A_i^c$$

4 Prove that for every two events A and B, the probability that exactly one of the two events will occur is

$$Pr(A) + Pr(B) - 2Pr(AB).$$

For every two events A and B, show that

$$Pr(A) = Pr(AB) + Pr(AB^c).$$

5 Let A_i, for $i = 1, 2, \cdots$, be an arbitrary infinite sequence of events and let B_j, for $j = 1, 2, \cdots$, be another arbitrary infinite sequence of events defined as follows: $B_1 = A_1$, $B_2 = A_1^c A_2$, $B_3 = A_1^c A_2^c A_3, \cdots$. Prove that

$$Pr\left(\cup_{i=1}^n A_i\right) = \sum_{i=1}^n Pr(B_i), \text{ for } n = 1, 2, \cdots$$

6 **a.** Prove for all positive integer n

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

b. Prove for all positive integer n

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$

c. Prove that for all positive integer n

$$\sum_{i=0}^n \binom{n}{i}^2 = \binom{2n}{n}.$$