HOMEWORK 10

STA 624.01, Applied Stochastic Processes
Spring Semester, 2008

Due: Fri. April 4th, 2008

Regular Problems

1 (Lawler 3.11) Let X_t be a countinuous-time birth-dan-death process with the birth rate $\lambda_n = 1 + (1/(n+1))$ and death rate $\nu_n = 1$. Is it process positive recurrent, null reccurent, or transient?

2 (Lawler 3.10) Suppose Q is the rate matrix for an irreducible continuous time Markov chain on the finite state space. Suppose π is the stationary distribution for the chain. Find the stationary distribution for the underlying discrete time Markov chain.

3 (Lawler 3.14) Consider a birth-and-death process with $\lambda_n = 1/(n+1)$ and $\nu_n = 1$. Show that the process is positive recurrent and give the stationary distribution.

4 Consider Yule process with the rate λ (i.e., it is a BD process with birth rate $x\lambda$ and death rate $\mu = 0$ for any state $x \in S$). Let $Y_x = \inf\{t : X_t = x\}$ for $x \geq 2$. Compute $E(Y_x)$.

Computer Problem: M/M/k queue (a) Suppose you spend five minutes observing the number of people in the queue and suppose $k = 1$. Let $\lambda = 15$ and $\nu = 10$ per 5 minutes. By breaking that five-minute-long time interval into very small intervals of 1 second, simulate the number of people in the queue. Also estimate the average time to stay the same number of people in the queue for each number 0 to 100.

(b) Suppose you spend five minutes observing the number of people in the queue and suppose $k = 4$. Let $\lambda = 5$ and $\nu = 6$ per 5 minutes. By breaking that five-minute-long time interval into very small intervals of 1 second, simulate the number of people in the queue. Also estimate the average time to stay the same number of people in the queue for each number 0 to 100.