HOMEWORK 5

STA 624.01, Applied Stochastic Processes
Spring Semester, 2008

Due: Friday, February 15th, 2008

Readings: Sections 2.1 of text

When giving numerical answers, please give results to four siginificant figures unless they are integer answers. So $1/2 = .5000$, and $1/80 = .01250$ for example. Also please box your numerical answers.

Regular Problems

1 Gambler's Ruin, part I Suppose that a gambler plays a fair game so that at each play of the game she loses a \$1 with probability 1/2, and gains a \$1 with probability 1/2. The gambler starts with \$32 and stops when she reaches \$100 or is out of money.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

2 Gambler's Ruin, part II Now suppose that the gambler is betting on red in American roulette. The chance of winning a \$1 is now 18/38 and losing a \$1 is 20/38.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

The moral of the Gambler's Ruin problem: a tiny change away from a fair game can have an enormous effect on the chances of walking away a winner.

3 Find the function $f(n)$, $n = 1, 2, \cdots, 10$ that satisfies

$$f(n) = \frac{1}{4}f(n-1) + \frac{3}{4}f(n+1), n = 1, 2, \cdots 9,$$

and $f(0) = 0$, and $f(1) = 1$.

4 The Fibonacci number F_n are defined by $F_1 = 1$, and $F_2 = 1$ and for $n \geq 2$ $F_n = F_{n-1} + F_{n-2}$. Find a formula for F_n by solving the difference equation.

Computer Problems

Simple Random Walk in $\mathbf{Z}^d$ Consider simple random walk on $\mathbf{Z}^1$, $\mathbf{Z}^2$, and $\mathbf{Z}^3$. (i.e., the transition probability $P(X_{n+1} = i + 1 | X_n = i) = P(X_{n+1} = i - 1 | X_n = i) = 1/2$ for $\mathbf{Z}^1$, and $P(X_{n+1} = i + e_j | X_n = i) = P(X_{n+1} = i - e_j | X_n = i) = 1/4$ for $\mathbf{Z}^2$, where e_j is the unit vector, and $P(X_{n+1} = i + e_j | X_n = i) = P(X_{n+1} = i - e_j | X_n = i) = 1/6$, where e_j is the unit vector, for $\mathbf{Z}^3$). For each of these, start at the origin and do the following. Estimate the expected distance aways from the origin after t steps, for t running from 1 to 100. Just use Euclidean distance, so in $\mathbf{Z}^3$, the distance of point (x, y, z) from the origin is $\sqrt{x^2 + y^2 + z^2}$. Conjecture a formula for this expected distance for $d = \{1, 2, 3\}$.