Practice problems for Master degree prob Common Exam

Summer 2007

DTMC

Question 1: Let X_n be a DTMC on a countable state Ω space and let $N(x) = \sum_{m=1}^{\infty} I_{\{X_m = x\}}$. Show $y \in \Omega$ is recurrent if and only if $E[N(y)|X_0 = y] = \infty$.

Question 2: Let X_n be an irreducible DTMC on a countable state Ω space. Show that if a state $x \in \Omega$ is recurrent, then all states in Ω are reccurent.

Question 3: Suppose we have a branching process that a parent has no offspring with probability 1/4 and 2 offspring with probability 3/4. What is the extinction probability?

Question 4: Consider a discrete time Markov chain on the state space $\Omega = \{0, 1, 2\}$ whose transition probability matrix at time n is

$$P(n) = \begin{cases} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} & \text{when } n \text{ is even} \\ \begin{pmatrix} 1/3 & 2/3 & 0 \\ 2/3 & 0 & 1/3 \\ 0 & 1/3 & 2/3 \end{pmatrix} & \text{when } n \text{ is odd} \end{cases}$$

Propose an alternative state space such that this chain is time homogeneous and write down the transition probability matrix. What is the probability that the chain is in state 2?

Question 5: Determine the classes and periodicity of the state $\{0, 1, 2, 3\}$ for the Markov chain with a transition probability matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \end{pmatrix}.$$

Question 6: Do problems 1.1 thru 1.6, 1.14 (a, b, c, d), 1.17 from Chapter 1 and do 2.8, 2.9.