Name____

STA 624 Practice Exam 1

Applied Stochastic Processes

Spring, 2008

There are five questions on this test. DO use calculators if you need them. "And then a miracle occurs" is not a valid answer. There will be no bathroom break allowed. Please keep all prayers silent.

You have 50 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

Question 1: Definitions

(a) What does it mean for π to be a stationary distribution?

(b) What does it mean for π to be a limiting distribution?

(c) What does it mean for a state x to be recurrent?

(d) What does it mean for a state x to be transient?

(e) What does it mean for a state x to be aperiodic?

(f) What does it mean for a discrete time Markov chain to be irreducible?

(g) When is a stochastic process $X_1, X_2, \ldots$ a (time homegeneous) Markov chain?

(h) What is a branching process?

Question 2: True or False

Mark whether each of the following states is true (T) or false (F). State a reason for each question.

(a) An expected number of visit at a positive recurrent state is infinitely often.

(b) The expected time of visit at a null recurrent state is finitely often.

(c) The expected time between visits at a null recurrent state is finite.

(d) The expected time between visits at a positive recurrent state is finite.

(e) A probability to return a null recurrent state is 1.

(f) A probability to return a positive recurrent state is 1.

(g) All states in a finite state Markov chain can be transient.

(h) The limiting probability π_i is the probability of being in state i and infinite time has passed.

(i) The limiting distribution is a stationary distribution.

(j) A stationary distribution is a limiting distribution.

(k) If a discrete time Markov chain is irreducible and aperiodic, there exists a unique stationary distribution?

(l) All states in a discrete time Markov chain must be recurrent or transient.

Question 3: Theorems

(a) State the Ergodic Theorem for countable state space Markov chains.

(b) State the Ergodic Theorem for finite state space Markov chains.

Questions 4: Examples (Note: you do not have to prove that your example meets the required criteria, you just have to present it.)

(a) Give an example of a 4 state Markov chain with at least one transient communication class.

(b) Give an example of a 4 state Markov chain with period 3.

(c) Give an example of a null recurrent Markov chain.

(d) Give an example of a Branching process with the extinct probability equal to < 1.

(f) Give an example of a discrete time irreducible Markov chain which does not have the limiting distribution.

(g) Give an example of a discrete time Markov chain whose stationary distribution is not unique.

Question 5: Calculations

(a) Suppose we have a branching process that a parent has no offspring with probability 1/4 and 2 offspring with probability 3/4. What is the extinction probability?

(b) Determine the classes and periodicity of the state $\{0, 1, 2, 3\}$ for the Markov chain with a transition probability matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \end{pmatrix}.$$

(c) Consider the following transition matrices for a Markov chain on state space $\{0, 1, 2, 3\}$:

$$A = \begin{pmatrix} .2 & .2 & 0 & .6 \\ .3 & .1 & .3 & .3 \\ 0 & 0 & .5 & .5 \\ 0 & .4 & 0 & .6 \end{pmatrix}, A^5 \approx \begin{pmatrix} .0954 & .2487 & .1450 & .5110 \\ .0933 & .2470 & .1498 & .5100 \\ .0894 & .2448 & .1549 & .5110 \\ .0932 & .2487 & .1469 & .5112 \end{pmatrix}, A^{10} \approx \begin{pmatrix} .0929 & .2477 & .1486 & .5108 \\ .0929 & .2477 & .1486 & .5108 \\ .0929 & .2477 & .1486 & .5108 \\ .0929 & .2477 & .1486 & .5108 \end{pmatrix}$$

(1) If X_0 is uniform over states $\{0, 1, 2, 3\}$, then find $P(X_1 = 1)$.

(2) Find $P(X_{10} = 2, X_5 = 3 | X_0 = 1)$

(3) If this chain represents the number of people in a queue, then what is the long-term average number of people in the queue?

(d) Identify the recurrent, transient, and absorbing states in the following Markov chain on state space $\{0, 1, 2, 3, 4\}$.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1/4 & 1/4 & 1/3 & 0 & 1/6 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 3/4 \\ 0 & 0 & 0 & 5/6 & 1/6 \end{bmatrix}.$$

(1) Identify the classes in this chain and indicate the period of each class.

(2) What is the probability of realization $X_0 = 1, X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 0$ if the initial state distribution is uniform over the state space of this Markov chain?