HOMEWORK 0

STA624.01, Stochastic processes
Spring Semester, 2009

Due: Thurs January 15th, 2009

1 **(a)** For every two events A and B, show that

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c.$$

(b) For every three events A, B, C, show that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

and

$$A - (B \cap C) = (A - B) \cup (A - C)$$

2 **(a)** Prove that for every two events A and B, the probability that exactly one of the two events will occur is

$$Pr(A) + Pr(B) - 2Pr(AB).$$

(b) For every two events A and B, show that

$$Pr(A) = Pr(AB) + Pr(AB^c).$$

3 **(a)** Prove that for all positive integer n

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}.$$

(b) Prove for all positive integer n

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n.$$

(c) Prove for all positive integer n

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0.$$

4 We are interested in the probability that a patient has measles given the knowledge that they have spots:

$$Pr(\text{patient-has-measles}|\text{patient-has-spots}).$$

Sometimes we will know how likely some "evidence" is, if some hypothesis is true, but not the other way around. For example, we may know that 50% of people with measles have spots. We may also know that:

The only diseases that cause spots are measles, chickenpox and lassa fever. 60% of people with chickenpox have spots. 80% of people with lassa fever have spots. There is a 1% chance of someone in a given population having measles (given no evidence for or against). There is a 1% chance of them having chickenpox. There is a 0.05% chance of them having lassa fever. Calculate $Pr(\text{patient-has-measles}|\text{patient-has-spots})$.

5 **(a)** For any events A, B, and C, such that $Pr(C) > 0$, prove that

$$Pr(A \cup B|C) = Pr(A|C) + Pr(B|C) - Pr(AB|C).$$

(b) Prove the following statement:

Suppose A_1, A_2, and B are events such that $Pr(A_1B) > 0$. Then, A_1 and A_2 are conditionally independent given B if and only if $Pr(A_2|A_1B) = Pr(A_2|B)$.