HOMEWORK 1

STA624.01, Stochastic processes
Spring Semester, 2009

Due: Thus January 22nd, 2009

Most of this assignment is about basic probability and enough MATLAB so that I'm sure you've tried the program and are able to start it up. Under Course Documents you will find a basic guide to probability, and six tutorials on MATLAB.

For this assignment, you'll need to read the guide, the first tutorial in its entirety, and skim the third tutorial. (Later on you will need to know everything in the second tutorial if you just want to read in order. The remaining tutorials will not be required, they are just there for completeness if you want to learn more about MATLAB.)

1 Prove the following statement:

Let $A_1, \cdots, A_k$ be events such that $Pr(A_1 \cdots A_k) > 0$. Then, $A_1, \cdots, A_k$ are independent if and only if for every two disjoint subsets $\{i_1, \cdots, i_m\}$ and $\{j_1, \cdots, j_l\}$ of $\{1, \cdots, k\}$ we have

$$Pr(A_{i_1} \cdots A_{i_m} | A_{j_1} \cdots A_{j_l}) = Pr(A_{i_1} \cdots A_{i_m}).$$

2 Suppose that k evenets $B_1, B_2, \cdots, B_k$ form a partition of S. For $i = 1, \cdots, k$, let $Pr(B_i)$ be the prior probability of B_i. Let $A \subset S$ with $Pr(A) > 0$. Let $Pr(B_i|A)$ be the posterior probability of B_i given that the event A has occured. Prove that if $Pr(B_1|A) < Pr(B_1)$, then $Pr(B_i|A) > Pr(B_i)$ for at least one value of i.

3 Suppose that $X_1,\, X_2, \cdots X_n$ are independent identically distributed positive random variables with $\mathbb{E}(X_i) = \mu < \infty$ and $\mathbb{E}(X_i^{-1}) < \infty$. Let $S_n = \sum_{i=1}^n X_i$. Show that for $m \leq n$

$$\mathbb{E}(S_m/S_n) = m/n,$$

and for $n \leq m$

$$\mathbb{E}(S_m/S_n) = 1 + (m-n)\mu\mathbb{E}(S_n^{-1}).$$

4 Suppose that the random variables $X_1,\, X_2, \cdots X_n$ are independent identically distributed from a uniform distribution on the interval $[0, 1]$. Let $Y_1 = \min\{X_1,\, X_2, \cdots X_n\}$ and $Y_2 = \max\{X_1,\, X_2, \cdots X_n\}$. Find $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y_2)$. Show your work.

5 Suppose that $Var(X) = Var(Y)$ and $Var(X+Y) < \infty$ and $Var(X-Y) < \infty$. Show that $X+Y$ and $X-Y$ are uncorrelated.

Computer Problems

$$A = \begin{bmatrix} 0 & .9 & 0 & .1 \\ .1 & .2 & .2 & .5 \\ .7 & .1 & .1 & .1 \\ 0 & .5 & .3 & .2 \end{bmatrix}$$

and $x^T = [\ 0 \quad .3 \quad .5 \quad .2\ ]$.

(a) Evaluate $x^T A$.

(b) Find the eigenvalues of A.