HOMEWORK 3

STA 624.01, Applied Stochastic Processes
Spring Semester, 2009

Due: Thursday, February 19, 2009

Readings: Chapter 2 of text

Note: the computer problems require simulation and the use of a computer. You are allowed (encouraged, even) to use a computer in solving the other problems as well.

When giving numerical answers, please give results to four siginificant figures unless they are integer answers. So $1/2 = .5000$ for example. Also box your numerical answers.

Regular Problems

1 Do the problem 1.9.

2 Do the problem 1.10.

3 Do the problem 1.12.

4 Do the problem 1.14.

5 Do the problem 1.17.

Computer Problems
Random walk with reflecting boundaries

For this problem, please print out all code used and all results.

Recall we did a simulation with a random walk with reflecting boundaries such that: The state space is $\{1, 2, \ldots, n\}$. It is defined as follows:

$$\begin{aligned} P(X_{t+1} = i+1 | X_t = i) &= p, \ \forall i \in \{2, \ldots, n-1\} \\ P(X_{t+1} = i-1 | X_t = i) &= 1-p, \ \forall i \in \{2, \ldots, n-1\} \\ P(X_{t+1} = n-1 | X_t = n) &= 1-p \\ P(X_{t+1} = n | X_t = n) &= p \\ P(X_{t+1} = 1 | X_t = 1) &= 1-p \\ P(X_{t+1} = 2 | X_t = 1) &= p. \end{aligned}$$

Using the code you implemented do the following simulation:

a) For $n = 5, 6, 7, 8, 9, 10$ and for $p = 0.2, 0.5, 0.6$, estimate the expected number of steps needed to move to state n starting from state 1 in the Markov chain and make a plot.

b) For $n = 5, 6, 7, 8, 9, 10$ and for $p = 0.2, 0.5, 0.6$, estimate the expected number of steps needed to move to state n starting from state 3 in the Markov chain and make a plot.

c) If you change the transition probability at the boundaries such that:

$$\begin{aligned} P(X_{t+1} = n-1 | X_t = n) &= 0 \\ P(X_{t+1} = n | X_t = n) &= 1 \\ P(X_{t+1} = 1 | X_t = 1) &= 1 \\ P(X_{t+1} = 2 | X_t = 1) &= 0. \end{aligned}$$

For $n = 5, 6, 7, 8, 9, 10$ and for $p = 0.2, 0.5, 0.6$, estimate the expected number of steps needed to move to state n or 0 starting from state $\lfloor n/2 \rfloor$ in the Markov chain and make a plot.