HOMEWORK 0

STA624.01, Stochastic processes
Spring Semester, 2010

Due: Tue January 26th, 2010

1 The following probability mass function for the discrete random variable X defines a geometric distribution:

$$f(j) = p(1-p)^j, \ j = 0, 1, 2, \cdots, \ 0 < p < 1.$$

(a) Show that the probability generating function (p.g.f) of X satisfies

$$g_X(t) = p(1-(1-p)t)^{-1}.$$

(b) Use the p.g.f to find the mean μ_X and variance σ_X^2.

2 The continuous random variables X_1 and X_2 have the joint probability density function (p.d.f)

$$f(x_1, x_2) = e^{-x_1 - x_2}, \ 0 < x_1 < \infty, \ 0 < x_2 < \infty$$

and zero otherwise.

(a) Show that X_1 and X_2 are independent and have exponential distributions.
(b) Find the moment generating function (m.g.f.) of X_1 and X_2.
(c) Find $E(e^{t(x_1+x_2)})$.

3 Suppose X_1 and X_2 are independent, continurous random variables with the joint p.d.f. satisfying $f(x_1)f(x_2)$.

(a) Show that

$$E(X_1 X_2) = E(X_1)E(X_2).$$

(b) For the joint p.d.f

$$f(x_1, x_2) = e^{-x_1 - x_2}, \ 0 < x_1 < \infty, \ 0 < x_2 < \infty$$

find $E(X_1 X_2)$ and $E(X_1^2 X_2^2)$.

4 Suppose that the random variable X is exponentially distributed. Show that X has the following property:

$$P(X \geq t + \Delta t | X \geq t) = P(X \geq \Delta t).$$

5 **(a)** Suppose that X_1, $X_2, \cdots X_n$ are independent identically distributed positive random variables with $E(X_i) = \mu < \infty$ and $E(X_i^{-1}) < \infty$. Let $S_n = \sum_{i=1}^n X_i$. Show that for $m \leq n$

$$E(S_m/S_n) = m/n,$$

and

$$E(S_m/S_n) = 1 + (m-n)\mu E(S_n^{-1}).$$

(b) Suppose that the random variables X_1, $X_2, \cdots X_n$ are independent identically distributed from a uniform distribution on the interval $[0,1]$. Let $Y_1 = \min\{X_1, X_2, \cdots X_n\}$ and $Y_2 = \max\{X_1, X_2, \cdots X_n\}$. Find $E(Y_1)$ and $E(Y_2)$. Show your work.