HOMEWORK 1

STA 624.01, Applied Stochastic Processes
Spring Semester, 2010

Due: Thursday, February 25th, 2010

Readings: Chapter 2 of text,

Note: the computer problems require simulation and the use of a computer. You are allowed (encouraged, even) to use a computer in solving the other problems as well.

When giving numerical answers, please give results to four siginificant figures unless they are integer answers. So $1/2 = .5000$ for example. Also box your numerical answers.

Regular Problems

1 Gambler's Ruin, part I Suppose that a gambler plays a fair game so that at each play of the game she loses a \$1 with probability 1/2, and gains a \$1 with probability 1/2. The gambler starts with \$32 and stops when she reaches \$100 or is out of money.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

2 Gambler's Ruin, part II Now suppose that the gambler is betting on red in American roulette. The chance of winning a \$1 is now 18/38 and losing a \$1 is 20/38.

a) What is the probability that the gambler ends up with \$100?

b) What is the expected number of plays before the gambler has either \$100 or \$0?

c) Now suppose the gambler starts with \$73. What is the probability that the gambler ends up with \$100?

3 Suppose that two unbiased coins are tossed repeatedly and after each toss the accumulated number of heads and tails that have appeared on each coin is recorded. Let X_n be the difference in the accumulated number of heads on coin A and coin B after the nth toss, i.e., $X_n =$ (Total number of heads on coin A) $-$ (Total number of heads on coin B). Thus, the state space $S = \{0, \pm 1, \pm 2, \cdots\}$. Show that the zero state, where the total number of heads equal on each coin, is null recurrent.

4 (Lawler 2.3) Consider the Markov chain with state space $\Omega = \{0, 1, 2, \ldots\}$ and transition probabilities

$$p(x, x + 1) = 2/3;\ p(x, 0) = 1/3.$$

Show that the chain is positive recurrent and give the limiting probability π.

5 (Lawler 2.4) Consider the Markov chain with state space $\Omega = \{0, 1, 2, \ldots\}$ and transition probabilities

$$p(x, x + 2) = p,\ p(x, x - 1) = 1 - p,\ x > 0.$$

$$p(0, 2) = p,\ p(0, 0) = 1 - p.$$

For which values of p is this a transient chain?

Computer Problems

Simple Random Walk in $\mathbf{Z}^d$ Consider simple random walk on $\mathbf{Z}^1$, $\mathbf{Z}^2$, and $\mathbf{Z}^3$. (i.e., the transition probability $P(X_{n+1} = i+1|X_n = i) = P(X_{n+1} = i-1|X_n = i) = 1/2$ for $\mathbf{Z}^1$, and $P(X_{n+1} = i+e_j|X_n = i) = P(X_{n+1} = i-e_j|X_n = i) = 1/4$ for $\mathbf{Z}^2$, where e_j is the unit vector, and $P(X_{n+1} = i+e_j|X_n = i) = P(X_{n+1} = i-e_j|X_n = i) = 1/6$, where e_j is the unit vector, for $\mathbf{Z}^3$). For each of these, start at the origin and do the following. Estimate the expected distance aways from the origin after t steps, for t running from 1 to 100. Just use Euclidean distance, so in $\mathbf{Z}^3$, the distance of point (x, y, z) from the origin is $\sqrt{x^2+y^2+z^2}$. Conjecture a formula for this expected distance for $d = \{1, 2, 3\}$.

Extra Credits (2 points): Write breif summary of Dr Peter Huggin's talk on Feb 22nd, Mon.

(1) Describe what his problem is.
(2) Describe methods what he uses.
(3) What are benefits over other methods?
(4) What are difficulties?