HOMEWORK 3

STA 624.01, Applied Stochastic Processes
Spring Semester, 2010

Due: Thursday, March 25th, 2010

Readings: Section 3.1 and 3.2 of text

Regular Problems

1 (Lawler 2.8) Given a braching process with the following offspring distributions, determine the extinction probability a:

(a)

$$p_0 = 0.25, \; p_1 = 0.4, \; p_2 = 0.35.$$

(b)

$$p_0 = 0.5, \; p_1 = 0.1, \; p_3 = 0.4.$$

(c)

$$p_0 = 0.91, \, p_1 = 0.05, \, p_2 = 0.01, \, p_3 = 0.01, \, p_6 = 0.01, \, p_13 = 0.01.$$

2 (Lawler 2.9) Consider the branching process with

$$p_0 = 0.5, \; p_1 = 0.1, \; p_3 = 0.4.$$

Suppose $X_0 = 1$.

(a) what is the probability that the population is extinct in the second generation ($X_2 = 0$), given that it did not extinct in the first generation ($X_1 > 0$)?

(b) what is the probability that the population is extinct in the third generation ($X_3 = 0$), given that it did not extinct in the second generation ($X_2 > 0$)?

3 Suppose X_n is a branching process with $E(\xi) = \mu < 1$. Let $Z = \sum_{n=0}^{\infty} X_n$. Suppose $X_0 = 1$. Show that

$$E[Z] = 1/(1-\mu).$$

4 Let $\xi_1, \xi_2, \cdots$ be a sequence of iid random variables. Let N be a discrete random variable and independent of $\xi_1, \xi_2, \cdots$. Define a discrete random variable X as

$$X = \begin{cases} 0 & \text{if } N = 0 \\ \xi_1 + \xi_2 + \cdots + \xi_N & \text{if } N \neq 0. \end{cases}$$

Prove that

$$Var(X) = \mathbb{E}(N)\sigma^2 + \mu^2 Var(N)$$

where $\mu = \mathbb{E}(\xi_i)$ and $\sigma^2 = Var(\xi_i)$.

Computer Problems

Branching process (Lawler 2.12) Consider the branching process with

$$p_0 = 1/3, \; p_1 = 1/3, \; p_2 = 1/3.$$

With the aid of computer, find the probability that the population dies out after n steps where $n = 20, 100, 200, 1000, 1500, 2000, 5000$. Do the same with the values

$$p_0 = 0.35, \; p_1 = 0.33, \; p_2 = 0.32.$$

Then do the same with with the values

$$p_0 = 0.32, \; p_1 = 0.33, \; p_2 = 0.35.$$