HOMEWORK 4

STA 624.01, Applied Stochastic Processes
Spring Semester, 2010

Due: Thursday, April 8th, 2010

Readings: Section 3.3 of text

Regular Problems

1 Measurements in a telephone exchange show that, during business hours, a telephone call in progress ends with constant rate $\lambda = 0.5$ per minute.

(a) What is the distribution of the duration of a call?

(b) What is the probability that a call lasts no longer than one minute?

(c) What is the probability that a call that has lasted for one minute already, will be finished during the next minute?

2 (a) Consider N independent Poison processes, where the rate of the ith Poison process is λ_i for $1 \le i \le N$. What is the probability that the first arrival from any process belongs to the jth process?

(b) Bus riders arrive at a bus stop as a Poisson process with its rate λ. They wait for the $i \in \{\text{blue, red, green}\}$ line with probability p_i. Line i buses arrive as a Poisson process with its rate λ_i and pick up all passengers waiting for bus i. Construct a CTMC model of this scenario. Also list all possible infinitesimal rates. **Hint**: There are three processes here and the state space is $\mathbb{Z}_+^3$ where $Z_+ := \{0, 1, 2, \cdots\}$.

3 Suppose $T_1, \cdots, T_n$ are independent randam variables each exponential with rates $b_1, \cdots, b_n$, respectively. Let $T = \min\{T_1, \cdots, T_n\}$. Show (in details) that

$$P(T_1 = T) = \frac{b_1}{b_1 + \cdots + b_n}.$$

4 In a road traffic survey the number of cars passing a market at a road was counted. The streams of cars in the two directions were a priori modelled as independent Poisson processes of the rates 2 and 3 per minute, respectively. It was decided to stop the counting once 400 cars has passed. Let T_{400} be that random time point and compute using appropriate approximations, a time T such that $P(T_{400} \le T) = 0.90$.

5 (Not by a group. Please each person write a summary by him/her self and submit a summary by each person.)

Go to a talk given by Dr Sturmfels on March 26th Fri at 3pm. Then summarize his talk as following:

- What is his mathematical problem?
- What is his approach to solve it?
- What is the main theorem?
- What aspect of his method is better than others?
- What is a weakness of his method?

Computer Problem This computer problem has three parts. In the first part we will look at the Poisson process, in the second we will look at the Poisson distribution, and in the third we will examine the relationship between the Poisson distribution and the binomial distribution.

(a) The Poisson distribution is often used to model the number of events that occur in a fixed time interval or fixed spatial region when they are occurring "at random", like shooting stars.

Suppose you spend five minutes watching shooting stars, and you know that on average they fall at 3 per minute. Thus, the rate parameter for our five-minute time period is lambda=15. We expect, on average, to see 15 shooting stars per five minutes. Show what a Poisson process might look like by using a simulation by breaking that five-minute-long time interval into very small intervals of 1 second and plot them.

(b) Simulate 10,000 star-gazing five-minute periods all at once. Then estimate the count of how many shooting stars you would see in a five-minute stretch. ALso estimate its mean, standard deviation, and variance.

(c) In class we learned that Binomial(n, p) can be approximated by Poisson(λ) with $\lambda = np$, when n is very large. Let us check the approximation. Try different values of $n = 75$ and $n = 100$ with the same λ in (a). See how the Poisson approximates a binomial better when n (in the binomial) is large. Compare the Binomial(n,p) plot against to the Poisson(lambda = np) plot.